Problem of the week

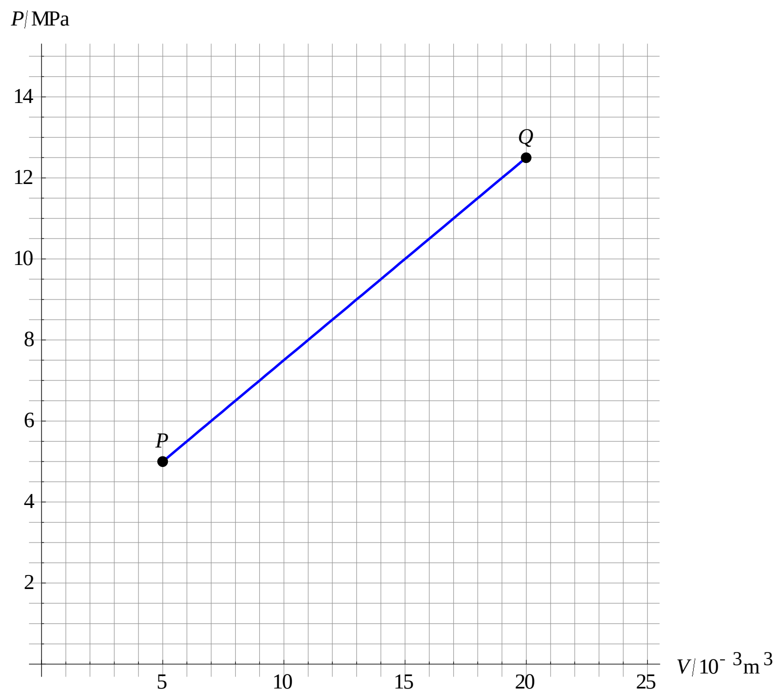
Thermodynamics

(a) State what is meant by the internal energy of

(i) an ideal gas,

(ii) a real gas.

(b) The graph shows the variation with volume of the pressure of an ideal monatomic gas. The temperature at P is 320 K.



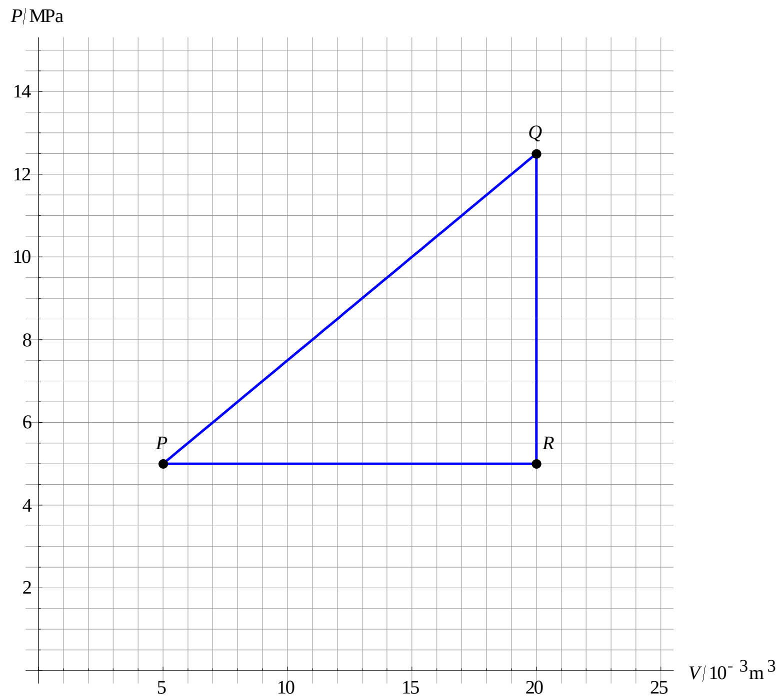
For the expansion from P to Q

(i) determine the work done by the gas,

(ii) calculate the change in internal energy,

(iii) show that the thermal energy supplied to the gas is 4.7×10^5 J.

(c) The gas in (b) undergoes the cycle PQRP as shown in the graph.

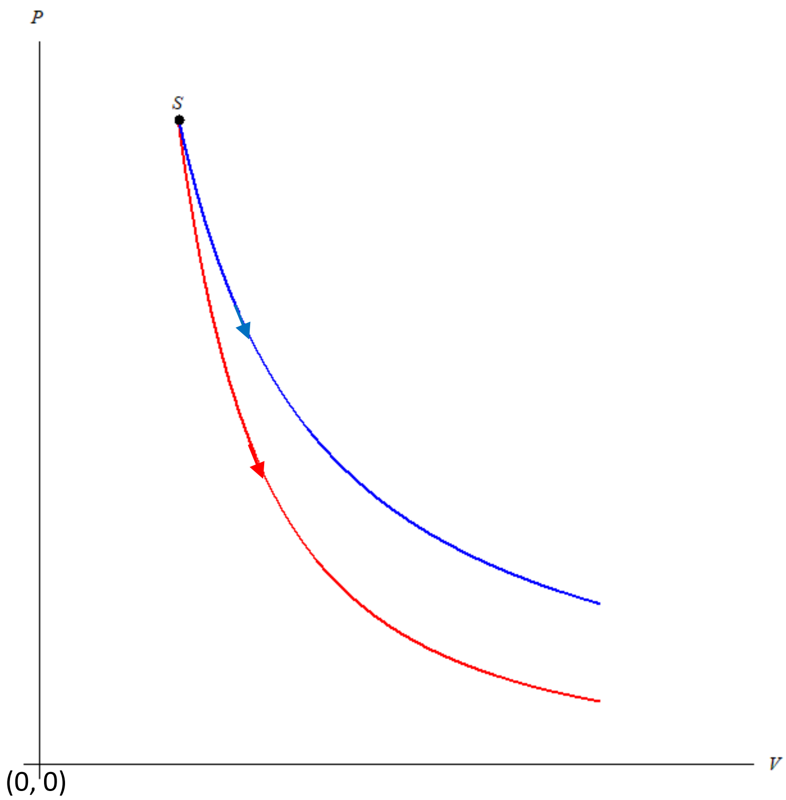


(i) Determine the efficiency of this cycle.

(ii) Estimate the Carnot efficiency for a heat engine working between the extreme temperatures of the cycle above.

(iii) Comment on the answer to (ii).

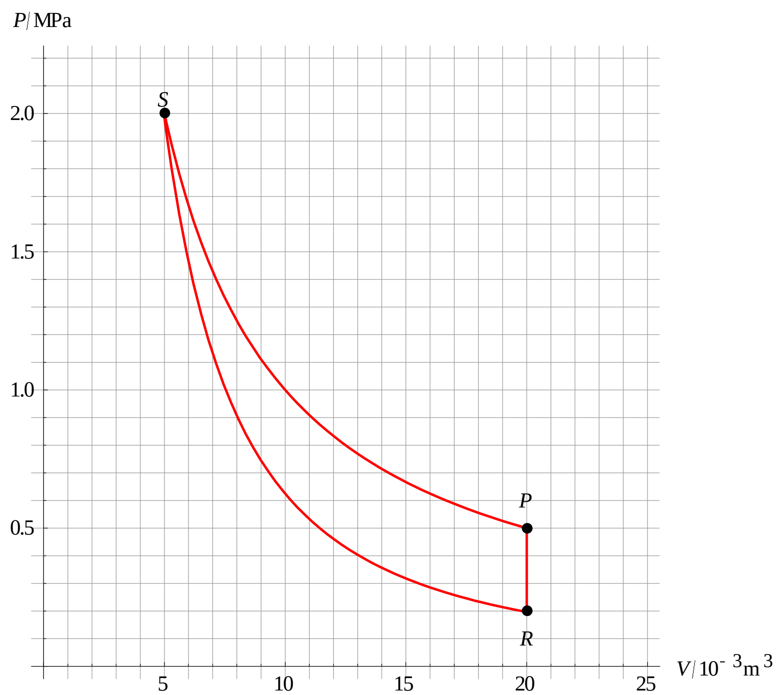
(d) The graph shows two expansions of the same ideal gas from an initial state S to final states with the same volume. One expansion is adiabatic and the other is isothermal.



Explain why the adiabatic curve is the red curve.

(e) A quantity of heat 1200 J is provided to an ideal gas at constant volume increasing the temperature by 5.0 K. Determine the heat that must be provided to the same quantity of another ideal gas at constant pressure so that the change in temperature is also 5.0 K. The gas does work 800 J in expanding.

(f) An ideal gas undergoes a cycle SPRS. SP is an isothermal and RS is an adiabatic. The temperature at S is 300 K. The gas expanding along SP does 1.2×10^5 J of work.



Determine the change in the entropy of the gas along PR.

Answers

(a)

(i) It is the total random kinetic energy of its molecules.

(ii) It is the total random kinetic energy of its molecules plus the total intermolecular potential energy.

(b)

(i) We need the area under the curve:

$$W = \frac{(5.0+12.5)\times 10^{6}}{2}\times(20-5.0)\times 10^{-3} = 1.312\times 10^{5} \approx 1.3\times 10^{5}\text{ J}.$$

(ii) $\Delta U = \frac{3}{2}Rn\Delta T = \frac{3}{2}(\frac{PV}{T})\Delta T$. The temperature at Q is

$$\frac{5.0\times 10^{6}\times 5.0\times 10^{-3}}{320} = \frac{12.5\times 10^{6}\times 20\times 10^{-3}}{T} \Rightarrow T = 3200\text{ K}.$$ Hence,

$$\Delta U = \frac{3}{2}\times\frac{5.0\times 10^{6}\times 5.0\times 10^{-3}}{320}\times(3200-320) = 3.375\times 10^{5} \approx 3.4\times 10^{5}\text{ J}.$$

(iii) $Q = \Delta U + W = 3.375\times 10^{5} + 1.312\times 10^{5} = 4.687\times 10^{5} \approx 4.7\times 10^{5}\text{ J}$.

(c)

(i) Heat in is $Q = 4.687\times 10^{5}$ J. The net work is the area of the loop i.e.

$$W = \frac{(12.5-5.0)\times 10^{6}\times(20-5.0)\times 10^{-3}}{2} = 5.625\times 10^{4}\text{ J}$$ and so the efficiency is

$$\frac{5.625\times 10^{4}}{4.687\times 10^{5}} = 0.12.$$

(ii) The highest and lowest temperatures of the cycle are 3200 K and 320 K. The Carnot efficiency for these temperatures is $1-\frac{320}{3200} = 0.90$.

(iii) The efficiency of the cycle is less than the Carnot efficiency as required by the second law of thermodynamics.

(d) In an isothermal expansion the temperature stays constant and in an adiabatic the temperature decreases ($Q = \Delta U + W$, $Q = 0$ and $W > 0$ so $\Delta U < 0$). Assume blue is adiabatic and red is isothermal. Then the final temperature for blue must be less than that for red. Comparing the final states, we see that the red curve has lower temperature than the blue because it has lower pressure at the same volume. This is a contradiction and so, the assumption that blue is adiabatic is false. The adiabatic is the red curve.

(e) For the constant volume case $Q = \Delta U + W = \Delta U + 0$. Since the temperature change is to be the same the change in internal energy must be the same and so $Q' = \Delta U + W = Q + W = 1200 + 800 = 2000$ J.

(f) Along the entire cycle the change in entropy is zero. Along SP, $Q = \Delta U + W = 0 + 1.2\times10^5$ J. Hence the change in entropy along SP is $\frac{1.2\times10^5}{300} = +400$ J K^{-1}. Along the adiabatic $\Delta S = 0$ because Q = 0. Hence, along PR, $\Delta S = -400$ J K^{-1}.